PROBABILITY OF FAILURE – LOAD AND RESISTANCE FACTORS

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Abstract: In the new generation of design code, safety of structures is provided in form of load and resistance factors. Safety is measured in terms of the reliability index. The acceptability criterion in the selection of load and resistance factors is closeness to the target reliability index which can be different depending on limit state. The paper presents a procedure to determine these factors using the concept of „design point". The coordinates of design point are equal to factored load or factored resistance. The required input data includes for each load component and resistance: mean values, bias factor (ratio of mean to nominal), standard deviation or coefficient of variation. The procedure is demonstrated on example of bridge design code (AASHTO) for prestressed concrete girders, guide for evaluation of existing bridges (AASHTO), also prestressed concrete girders, and design code for concrete buildings (ACI 318) – reinforced beams in flexure.

Keywords: load factor, resistance factor, design point, reliability index, dead load, live load, resistance, statistical parameters, bias factor, coefficient of variation

1. Introduction

A new generation of design codes and guides for evaluation of existing structures is based on consideration of limit states and failure scenarios. Contrary to the allowable stress design (ASD) or working stress design (WSD), safety margin is represented by load and resistance factors. However, the principle role of design codes and guides is to make sure that the designed or evaluated structures perform their function with adequate reliability. It is impossible to eliminate the possibility of failure but the probability of failure should be kept at an acceptably low level. There are three important questions:
– How to measure safety?
– What is acceptable and what is unacceptable safety level?
– How to implement the acceptable safety in engineering practice?

There is a considerable literature on structural safety and reliability. Safety can be measured in terms of the reliability index, β. The acceptability/unacceptability criteria can then be expressed as a target reliability index, β_T. For each limit state, the selection of β_T depends on the consequences of failure and marginal cost of safety, and it may involve a higher or lower degree of subjective judgement. However, the objective of this paper is to focus on the last bullet item, i.e. implementation of the target reliability index in design codes and guides for evaluation of existing structures. In particular, the paper deals with derivation of load and resistance factors as coordinates of the so called „design point" (Nowak and Collins 2013).

2. Limit State Function and Reliability Index

For each limit state, a structural component can be in two states: safe when resistance, R, exceeds the load, Q, and unsafe (failure) when load exceeds resistance. The boundary between safe and unsafe states can be represented by the limit state function, in a simple form such as:

$$g = R - Q = 0 \quad (1)$$

Since R and Q can be considered as random variables, the probability of failure, P_f, is equal to probability of g being negative,

$$P_F = P\ (g < 0) \quad (2)$$

In general, R and Q can be functions of several variables such as dead load, live load, dynamic load, strength of material, dimensions, girder distribution factors, and so on. Therefore, the limit state function can be a complex function:

$$g(X_1, \ldots, X_n) = 0 \quad (3)$$

A direct calculation of probability of failure can be difficult, in particular when g is non-linear. Instead, reliability index, β, can be calculated and the relationship between, β, and the probability of failure, P_f, is as follows:

$$P_f = \Phi\ (-\beta) \quad (4)$$

and

$$\beta = -\ \Phi^{-1}(P_f) \quad (5)$$

where:

Φ – cumulative distribution function of the standardized normal random variable;
Φ^{-1} – the inverse of Φ (Nowak and Collins 2013).

There are several formulas and analytical procedures available to calculate β. If the limit state function is linear, and all the variables are normal (Gaussian), i.e.

$$g\ (X_1, \ldots, X_n) = a_0 + \sum_{i=1}^{n} a_i X_i \quad (6)$$

then

$$\beta = \frac{\mu_g}{\sigma_g} \quad (7)$$

where:

$$\mu_g = g(\mu_1, \ldots, \mu_n) \quad (8)$$

μ_i – mean value of X_i

$$\sigma_g = \sqrt{\sum (a_i\ \sigma_i)^2} \quad (9)$$

σ_i – standard deviation of X_i.

If the variables are non-normal, then Eq. 9 can be used as an approximation. Otherwise, more accurate value of β can be calculated using an iterative procedure or if the limit state function is non-linear, then accurate results can be obtained using Monte Carlo simulations (Nowak and Collins 2013).

3. Design Point

The result of reliability analysis is reliability index, β. In addition, the reliability analysis can be used to determine the coordinates of the „design point", i.e. the corresponding value of factored load for each load component and value of factored resistance. For the limit state function in Eq. 5, the design point is a point in n-dimensional space, denoted by (X_1^*, …, X_n^*), that satisfies Eq. 5, and if failure is to occur, it is the most likely combination of X_1^*, …, X_n^* (Nowak and Collins 2013).

For example, if the limit state function is given by Eq. 3, and R and Q are normal random variables, then the coordinates of the design point are (Nowak and Collins 2013):

$$R^* = \mu_R - \frac{\beta \sigma_R^2}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{10}$$

$$Q^* = \mu_Q + \frac{\beta \sigma_Q^2}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{11}$$

If R and Q are not both normally distributed then R^* and Q^* can be calculated by iterations. However, a relatively wider range of design point coordinates corresponds to the same value of reliability index, so in practice, Eq. 10 and Eq. 11 can be used even for non-normal distributions.

4. Load and Resistance Factors

Consider a load and resistance factor design (LRFD) formula,

$$\gamma_D D_n + \gamma_L L_n \leq \phi R_n \tag{12}$$

where: D_n – nominal dead load, γ_D – dead load factor, L_n – nominal live load, γ_L – live load factor, R_n – nominal resistance, ϕ – resistance factor.

Furthermore, $\gamma_D D$ is factored dead load, $\gamma_L L$ is factored live load and ϕR is factored resistance. But the coordinates of design point, D^*, L^* and R^* correspond to factored loads and factored resistance, respectively. Therefore, the load and resistance factors can be calculated as follows,

$$\gamma_D = D^* / D_n \tag{13}$$

$$\gamma_L = L^* / L_n \tag{14}$$

$$\phi = R^* / R_n \tag{15}$$

In many practical cases, to calculate load and resistance factors, the required input data for each load component and resistance includes the following: mean value, bias factor (mean-to-nominal ratio), λ, and standard deviation, σ, or coefficient of variation, V. The derivation of load and resistance factors is demonstrated for bridge design code AASHTO (2014), concrete buildings design code ACI 318 (2014), and a guide for evaluation of existing bridges (AASHTO (2011).

5. Bridge Design Code

The basic load combination for bridge load components include dead load, D, dead load due to the wearing surface, D_W, live load, L, and dynamic load, I. Each random variable is described by its cumulative distribution function (CDF), including the mean and standard deviation. It is also convenient to use the bias factor which is the ratio of mean-to-nominal value, denoted by λ, and the coefficient of variation, V, equal to the ratio of the standard deviation and the mean. Both λ and V are non-dimensional.

The total load is a sum of $D + Dw + (L + I)$. The statistical parameters of dead load that were used in the original calibration have not been challenged so far (NCHRP Report 368). Therefore, for factory-made components (structural steel and precast/prestressed concrete) $\lambda = 1.03$ and $V = 0.08$. For the cast-in-place concrete, $\lambda = 1.05$ and $V = 0.10$. For the wearing surface it is assumed that the mean thickness is 3.5 in (90 mm) with $\lambda = 1.00$ and $V = 0.25$. The statistical parameters for live load are taken from the recent SHRP2 R19B report (2015). The ratio of mean-to-nominal value, or bias factor for live load moment, is plotted vs. span length in Fig. 1 for the average daily truck traffic (ADTT) from 250 to 10,000. The mean value of the dynamic load factor is taken as 0.10 and the coefficient of variation for static and dynamic live load is taken as 0.14 (NCHRP Report 368). The total load as a sum of several components can be considered as a normal random variable. The load carrying capacity is considered as a product of three factors representing the uncertainties involved in material properties, dimensions/geometry and the analytical model. The statistical parameters, bias factor, $\lambda = 1.05$, and coefficient of variation, $V = 0.075$, that were used in the original calibration.

The dead load factor γD corresponding to the design point is shown in Fig. 2. The calculated live load factors are shown in Fig. 3. For most cases, the optimum live load factor γL is between 1.4 and 1.55 for ADTT = 10,000 and the range is 1.3 to 1.5 for ADTT = 250. Therefore, 1.55 can be considered as a conservative value of live load, even for ADTT = 10,000. The resistance factors are presented in Fig. 4 for prestressed concrete girders.

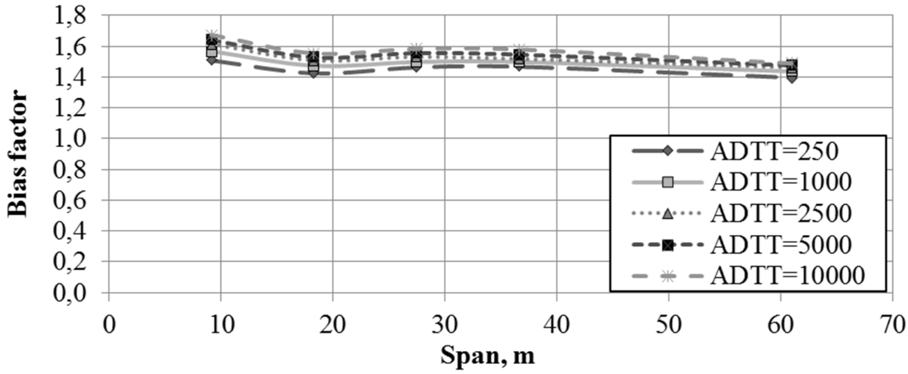

Fig. 1. Bias factor vs. Span length for the moment

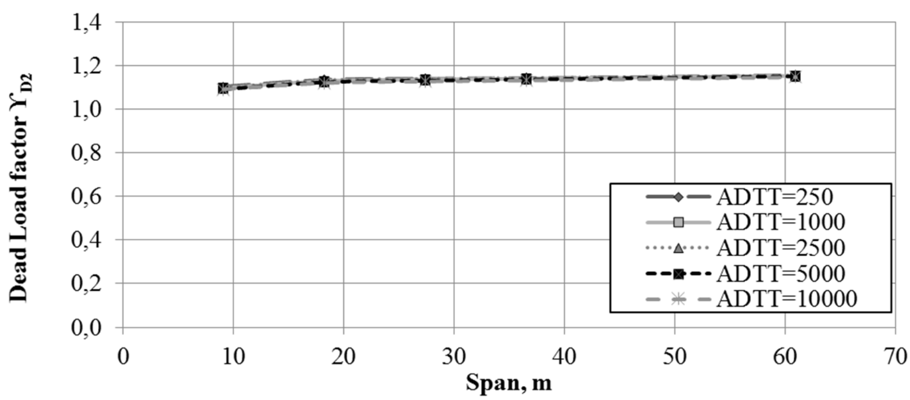

Fig. 2. Dead load factors vs. Span length for prestressed concrete girders

Therefore, the recommended new design formula is:

$$1.20\ (D+.\ D_w) + 1.6\ (L+I) < 0.85\ R \tag{13}$$

For comparison, the design formula in the current AASHTO (2014) is:

$$1.25\ (D+ 1.50\ D_w) + 1.75\ (L+I) < 1.00\ R \tag{14}$$

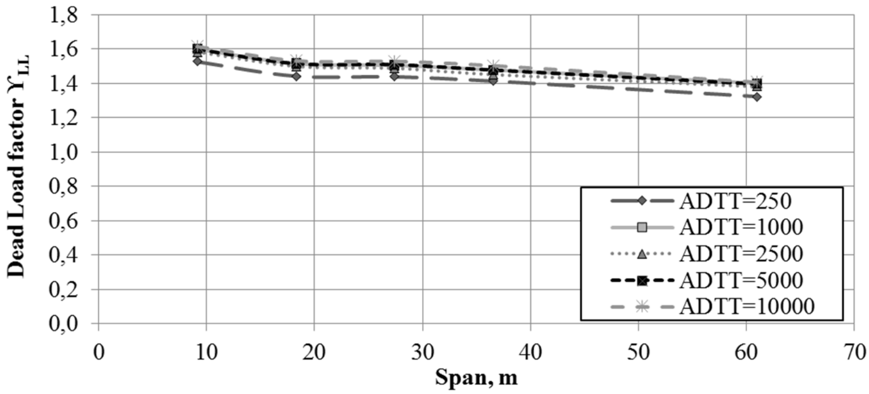

Fig. 3. Live load factor vs. Span Length for Prestressed Concrete Girders

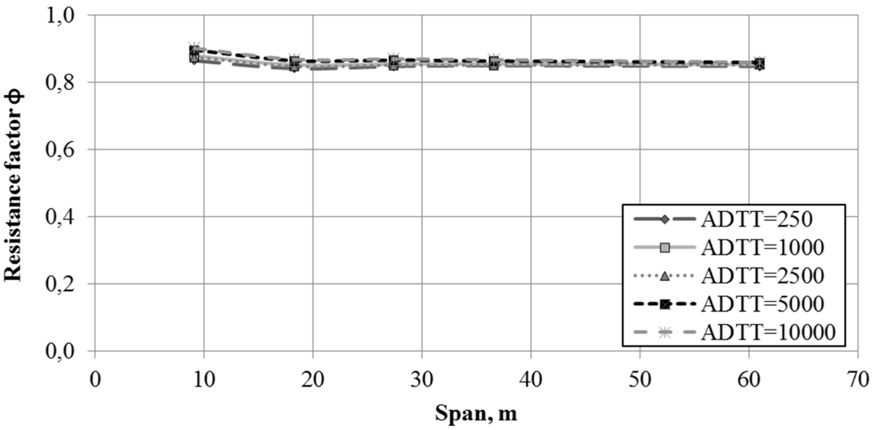

Fig. 4. Resistance Factor vs. Span Length for Prestressed Concrete Girders

6. Evaluation of Existing Bridges

Evaluation of existing bridges is governed by the Manual for Condition Evaluation of Bridges (AASHTO 2011). The statistical parameters for load and resistance are assumed the same as for new design. However, the target reliability index $\beta_T = 2.5$, while for new designs it is $\beta_T = 3.5$. The difference is due to cost of additional safety. To increase the reliability for a new design can be much less expensive than to upgrade an existing structure. Therefore, the load and resistance factors are calculated for prestressed concrete girders and $\beta_T = 2.5$. The results are shown in Fig. 5 for dead load factor, Fig. 6 for live load factor and Fig. 7 for resistance factor.

Therefore, the recommended formula specified for evaluation of existing bridges is:

$$1.20\ (D + D_w) + 1.4\ (L+I) < 0.90\ R \quad (15)$$

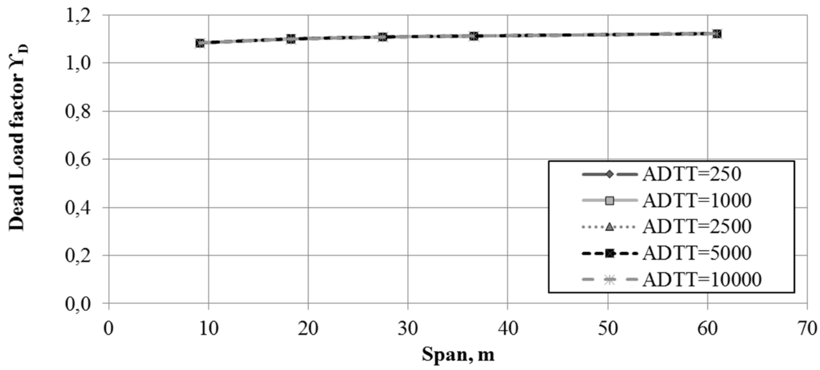

Fig. 5. Dead load factor for evaluation of existing bridges

For comparison, the formula specified in AASHTO is:

$$1.25\ D + 1.50\ D_w + 1.35\ (L+I) < 1.00\ R \quad (16)$$

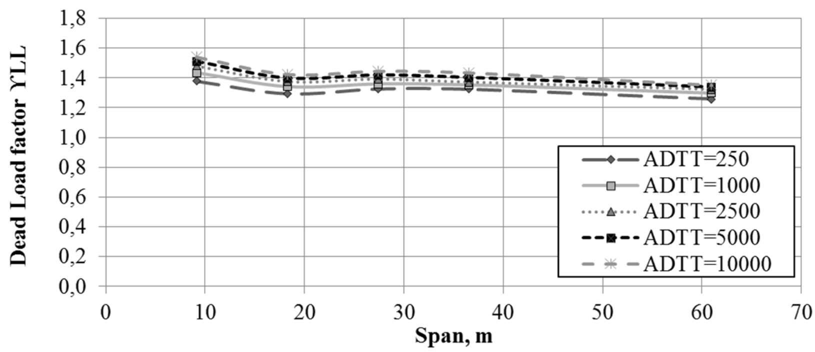

Fig. 6. Live load factor for evaluation of existing bridges

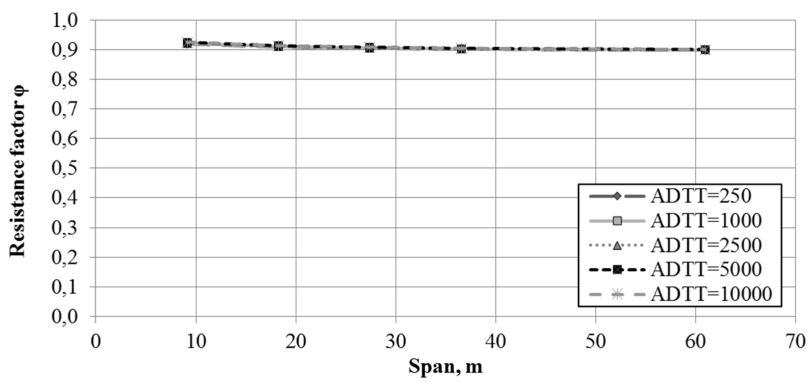

Fig. 7. Resistance factor for evaluation of existing bridges

7. Design Code for Concrete Structures

The statistical parameters for the design of reinforced concrete building structures can be deifferent than for bridges. The dead load can be assumed the same as listed for bridges. However, for live load the bias factor $\lambda = 1.00$ and coefficient of variation $V = 0.18$. For resistance representing moment carrying capacity of a reinforced concrete beam, two cases of reiforcement ratio are considered (Rakoczy and Nowak 2012) for $\rho = 0.60\%$, $\lambda = 1.10$ and $V = 0.12$. The resulting dead load factors are shown in Fig. 8, live load factors in Fig. 9 and reistance factors in Fig. 10. Two types of distribution are considered for resistance: normal and lognormal. The recommended load and resistance factors are,

$$1.2\,D + 1.6\,L < 0.9\,R \tag{17}$$

For comparison, the current ACI 318-14 design formula is:

$$1.2\,D + 1.6\,L < 0.9\,R \tag{18}$$

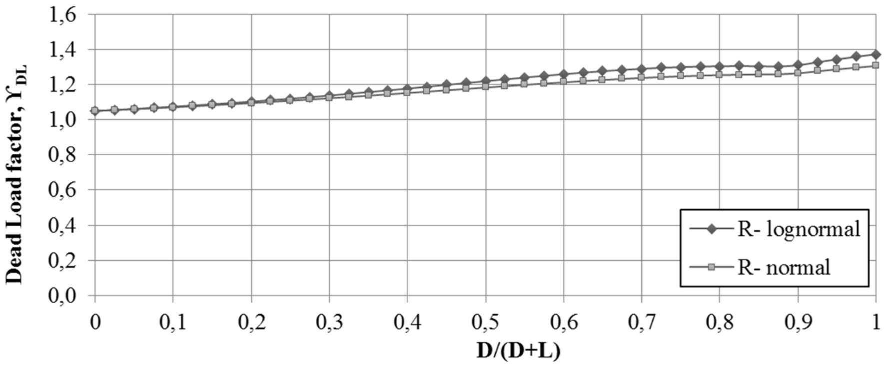

Fig. 8. Dead load factor for design of concete beams ρ = 0.6%

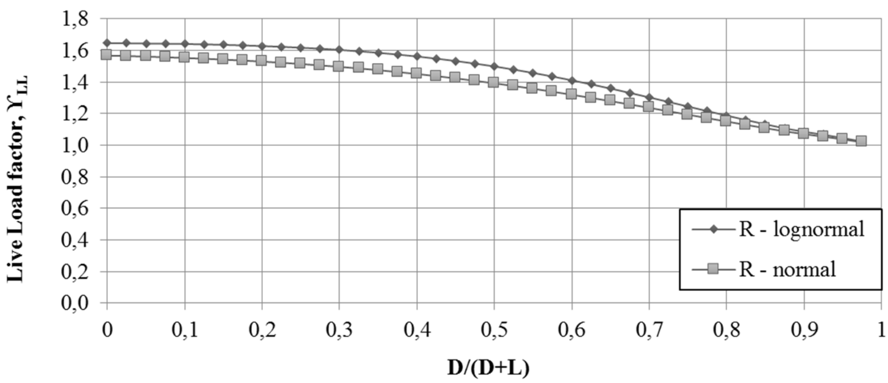

Fig. 9. Live load factor for design of concete beams $\rho = 0.6\%$

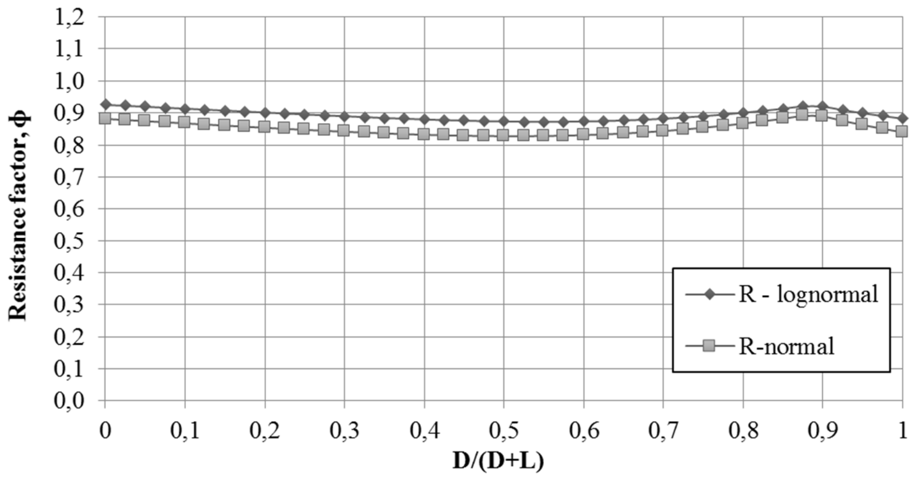

Fig. 10. Resistance factor for design of concete beams $\rho = 0.6\%$

8. Evaluation of Existing Concrete Structures

The statistical paramters of load and resistance for evaluation of exisitng reinforced concrete beams in buildings are assumed the same as for new design, however, the target reliability is taken $\beta_T = 2.5$, while for new designs it $\beta_T = 3.5$. The results are shown in Fig. 11 for dead load factor, Fig. 12 for live load factor and Fig. 13 for resistance factor.

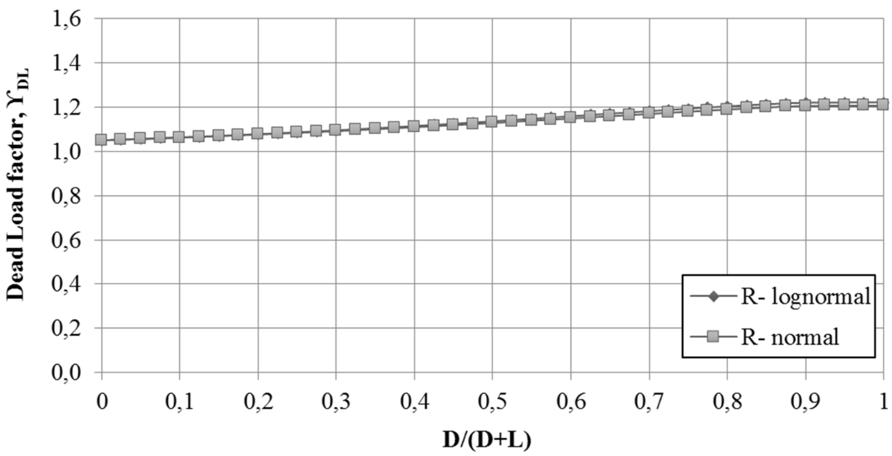

Fig. 11 Dead Load Factor for Evaluation of Existing of Reinforced Concrete Beams

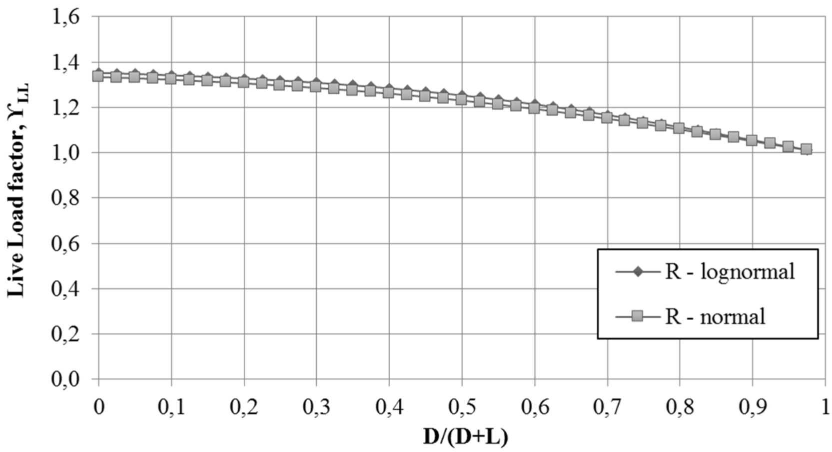

Fig. 12. Live Load Factor for Evaluation of Existing of Reinforced Concrete Beams

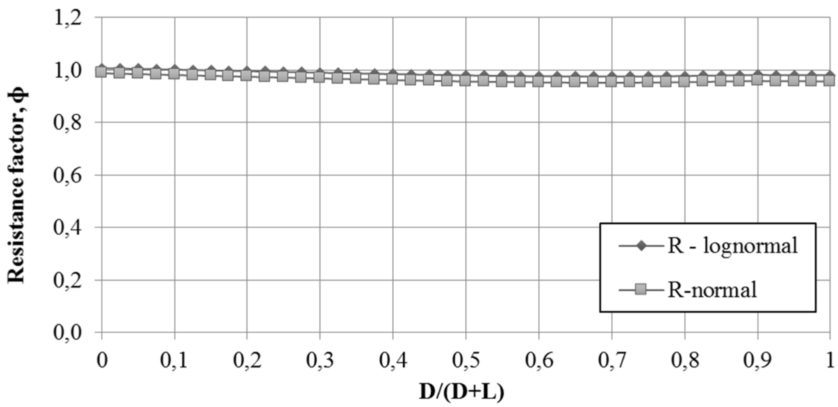

Fig. 13. Resistance Factor for Evaluation of Existing of Reinforced Concrete Beams

Therefore, the recommended formula specified for evaluation of existing reinforced concrete beams is:

$$1.2\ D + 1.35\ L < 0.95\ R \tag{19}$$

9. Conclusions

In the new generation reliability-based design codes, the load and resistance factors correspond to the coordinates of the design point. The presented procedure results in a set of load and resistance factors that provide a closer fit to the target reliability index.

The procedure is demonstrated on a prestressed concrete girder and a reinforced concrete beam. For both components, the load and resistance factors are derived for a new design and evaluation of existing structure.

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PRAWDOPODOBIEŃSTWO AWARII – WSPÓŁCZYNNIKI OBCIĄŻEŃ I NOŚNOŚCI

Streszczenie: Normy do projektowania konstrukcji oraz wytyczne do oceny istniejących konstrukcji mają zapewnić wymagany zapas bezpieczeństwa. Dlatego bardzo ważny jest wybór współczynników obciążeń i nośności konstrukcji. Głównym celem referatu jest przedstawienie procedury do obliczenia optymalnych współczynników obciążeń i nośności konstrukcji. Dla każdego składnika obciążenia, iloczyn współczynnika obciążenia oraz wartości nominalnej tego składnika obciążenia odpowiada najbardziej prawdopodobnemu przebiegowi awarii. Podobnie jest z iloczynem współczynnika nośności oraz wartości nominalnej nośności. Teoretycznie, optymalne współczynniki obciążenia i nośności mogą być różne dla różnych stanów granicznych, a nawet w zależności od materiałów, rozpiętości i innych parametrów. Ze względów praktycznych, współczynniki normowe są zaokrąglane a liczba różnych współczynników powinna być jak najmniejsza. Dlatego sprawdza się jak szeroki zakres konstrukcji może być adekwatny. Wyznaczanie współczynników obciążeniowych i nośności jest przedstawione na przykładzie normy mostowej do projektowania (AASHTO), normy do oceny istniejących mostów (AASHTO), oraz normy do projektowania budowli żelbetowych (ACI 318).

Słowa kluczowe: współczynnik obciążenia, współczynnik nośności, prawdopodobieństwo awarii, wskaźnik niezawodności, obciążenie stale, obciążenie zmienne, nośność konstrukcji, parametry statystyczne