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STANY GRANICZNE: NOŚNOŚĆ CZY UŻYTKOWALNOŚĆ?

ULTIMATE OR SERVICEABILITY LIMIT STATES?

Streszczenie Nowa generacja norm projektowych jest oparta na stanach granicznych. Współczynniki obciążeń i nośności są ustalane poprzez kalibracje i analizę niezawodności konstrukcji. Metody obliczania niezawodności zostały opracowane dla stanów granicznych nośności dlatego jest potrzeba wypracowania obliczania niezawodności dla stanów granicznych użytkowości. W referacie przedstawiono propozycje kalibracji na przykładzie stanu granicznego ugięć oraz rozwarcia rys (dekompresji).

Abstract New generation of design codes is based on limit states. Load and resistance factors are determined in the reliability-based calibration. However, the reliability analysis procedures are available for the ultimate limit states and, therefore, there is a need to develop practical tool for calibration of the serviceability limit states. The paper present such a procedure and it is illustrated for SLS-deflection and SLS-decompression.

1. Introduction

The notion of limit state is fundamental in the LRFD design code. A limit state is defined as the boundary between acceptable and unacceptable performance of the structure or its component. However, for any structure or structural component, there can be many different limit states. For example, the limit states for a prestressed concrete beam include the moment carrying capacity, shear capacity, torsion capacity, and also deflection, tensile stress at the bottom, and cracking among others.

The AASHTO code [1] is calibrated but for the strength (ultimate) limit states of moment carrying capacity and shear capacity [2] with the limit state function in the simple form,

$$g = R - Q \quad (1)$$

where R = resistance (load carrying capacity) and Q = load effect (sum of dead load, live load and dynamic load).

Both R and Q can be treated as random variables with the statistical parameters assessed from load survey, material tests, etc. It was assumed that R and Q are uncorrelated random variables. Furthermore, R was treated as constant in time and Q was calculated as the extreme expected value in the economic lifetime of bridge, i.e. 75 years. The major time varying load component is live load. The extreme 75 year live load was obtained by extrapolation of the distribution function obtained in the truck survey representing a two week heavy traffic.

The ultimate limit states (ULS) represent the boundary of the load carrying capacity [2] and acceptable performance. Exceeding the strength limit state results in a collapse or failure, an event that should not occur any time during the lifetime of the structure. Therefore, there is a need for an adequate safety margin expressed in form of a target reliability index, β_T . For bridge girders, the target reliability is taken as, $\beta_T = 3.5$ [3].

The service limit states (SLS) represent the boundary of desirable performance. When the SLS is exceeded, the result can be a need for repair or replacement of components, repeated exceeding of SLS can lead to deterioration and eventually collapse or failure (ULS). ULS absolutely cannot be exceeded as it can result in a total failure and collapse. In general, SLS can be exceeded but the frequency and magnitude have to be within limits.

Therefore, consideration of service limit states requires a different input data than ULS. In ULS, the limit state function is defined with two variables, R and Q (see Eq. 1), where R = resistance (constant in time) and Q = extreme load effect specified in the code. In case of SLS:

- The definition of resistance can be very difficult.
- Acceptable performance can be subjective (full life-cycle analysis is required).
- Resistance and load effects can be and often are correlated.
- Load is to be considered as a function of time, described by magnitude and frequency of occurrence.
- Resistance and loads can be strongly affected by quality of workmanship, operation procedures and maintenance
- Resistance can be a subject to changes in time, mostly but not only deterioration, with difficult to predict initiation time and time-varying rate of deterioration (e.g. corrosion, accumulation of debris, cracking)
- Resistance can depend on geographical location (climate, exposure to industrial pollution, exposure to salt as deicing or proximity to the ocean)

An example of the difficulties in the approach to SLS can be treatment of cracking in the design of prestressed precast concrete girders. The design codes limit occurrence of the tensile stress at the bottom of the girder. However, even for a properly designed girder, the probability of exceeding the tensile strength of concrete is very high. Under heavy traffic, there is 50% probability that the crack will open once every few weeks. Frequent opening of the crack can facilitate penetration of salt water (pumping action) and corrosion of prestressing strands. In the development of the Ontario Highway Bridge Design Code (Lind and Nowak 1978), predecessor to the AASHTO LRFD Code, it was decided that opening of the crack once every three weeks is acceptable, with the probability of 50%, but more often than that is not acceptable. Therefore, the limit state function was formulated as in Eq. 1, but with R = decompression moment for concrete and Q = maximum three week moment due to trucks, and in the design formula, R = mean decompression moment and Q = mean maximum three week moment. The mean values were used because of probability of occurrence = 50%, which corresponds to $\beta_T = 0$.

2. Ultimate Limit States

Examples of the ultimate limit states include:

- Moment carrying capacity of a beam
- Shear capacity of a beam
- Tensile strength of a cable
- Torsional capacity of a beam
- Overall bucking capacity of a column

- Local buckling of the flanges or web
- Loss of stability of the structure

The reliability analysis starts with the formulation of a limit state function also known as a performance function. In a special case, when the effect of all loads can be represented by one variable, Q , and the resistance of the structure can also be represented by one variable, R , then the limit state function, $g(R, Q)$, can be expressed as in Eq. 1. In a case when random variables R and Q are independent, the probability of failure, P_f , and reliability index, β , can be defined as:

$$P_f = \text{Prob}(R - Q < 0) = \int_{-\infty}^{+\infty} f_Q(x) \cdot F_R(x) dx \quad (2)$$

$$\beta = -\Phi^{-1}(P_f) \quad (3)$$

where Φ is the standard normal distribution function, $f_Q(x)$ is the probability density function of load and $F_R(x)$ is the cumulative function of resistance. In general, however, the limit state function is a function of many variables (load components, influence factors, resistance parameters, material properties, dimensions, analysis factors, etc.). Consequently, the computations using Eq. 2 become very complex. Therefore, the probability of failure can be determined indirectly by calculation of the reliability index, β , and then from Eq. 3:

$$P_f = \Phi(-\beta) \quad (4)$$

The available procedures vary with regard to accuracy, required input data and computing cost. Parameters of R and Q , or even the limit state function g , can also be obtained by Monte Carlo simulations [2].

3. Serviceability Limit States

The major service limit state problems are related to function and maintenance of the structure. The LRFD service limit states include limits on:

- live-load deflection,
- cracking of reinforced-concrete components,
- tensile stresses of prestressed-concrete components,
- compressive stresses of prestressed-concrete components,
- permanent deformations of compact steel components,
- slip of slip-critical friction bolted connections,
- settlement of shallow and deep foundations,
- gradual degradation of steel (corrosion)
- gradual degradation of concrete (alka-silica reaction)

among others. Some of these service limit states may relate to a specified design life; others do not. Many are presently very deterministic, such as some owners' wish to limit the tensile stresses in prestressed-concrete components to ensure a crack-free component. This service limit state could be calibrated to achieve a certain probability of a crack-free component, but this calibration includes a service life only in the determination of the live-load the component must resist, for example, a 75-year live load.

For bridges, the serviceability can be affected by bridge bearings, joints, water drainage and steel coating. The current practice indicates that their performance can be strongly dependent on:

- Design parameters (dimensions, material properties, connections)
- Type and model (joint, bearing, drainage system, steel coating)
- Location (winter/freeze-and-thaw cycles, urban/rural, industrial pollution, exposure to salt water)
- Traffic volume and magnitude
- Quality of workmanship (construction, operation, maintenance)
- Correlation between bearing, joint, drainage system, coating (no-joint, leaking joint) and other parameters.

The designer has control over the first two items (design parameters and selection of the type and model). Based on the past practice, the designer can make assumptions with regard to location characteristics and traffic parameters. However, the prediction of the quality of workmanship involves a considerable degree of uncertainty, and yet it has a significant impact on the long term performance. The last item, the development of correlations, requires a considerable data base. In the proposed study, the available information will be collected and utilized to develop the interaction models for the considered items and parameters.

The formulation of the limit state function for SLS is much more complex than for the ULS. In many cases, the occurrence of load exceeding the resistance is not unaccepted as long as it does not occur frequently. Therefore, the frequency of occurrence, or return period, has to be considered. If the limit state function can be formulated in terms of load and resistance, the actual limit state can be the critical frequency of occurrence or critical return period.

4. Target Reliability for ULS vs. SLS

The reliability analysis can be performed for both ULS and SLS using similar procedure and formulas. This analysis will lead to determination of the probability of exceeding the formulated limit state. For ULS, the acceptable probability level is very small, as failures are not tolerated during the economic life time of the structure. For SLS, the probability can be as high as 50% for some limit states, because it is more a question of return period (or frequency of occurrence) rather than the load exceeding the resistance.

In general, the consequences of exceeding SLS are an order or even orders of magnitude smaller than those associated with ULS. Therefore, an acceptable probability of exceeding a SLS is much higher than for ULS. If the target reliability index for ULS is $\beta_T = 3.5$ to 4.0 , then for SLS, $\beta_T = 0$ to 1.0 .

The load and resistance factors are determined in the calibration process, with the objective of maintaining closeness to the target reliability index. The calibration procedure for Serviceability Limit States (SLS) is different than for Ultimate Limit States (ULS). For ULS the procedure was presented in the available literature, e.g. [4] or [2]. For SLS, the procedure will be demonstrated for SLS-deflection and SLS-decompression for prestressed concrete girders.

5. Calibration for SLS-Deflection

For SLS, the limit state function can be formulated as shown in Eq. 1. For the SLS of deflection, R can represent the maximum acceptable deflection, and Q can represent the deflection caused by loads. The corresponding design formula can have a format similar to

$$\gamma L (L_n + I_n) \leq \phi R_n \quad (5)$$

where L_n and I_n are deflections due to nominal live load and nominal dynamic load, respectively, and R_n represents the maximum allowable nominal deflection. Load and resistance factors are selected so that the corresponding reliability index is close to the target value, or that the probability of exceeding the maximum allowable deflection does not exceed the target value. However, it is acceptable that the maximum allowable deflection be exceeded during the life time of the bridge (75 years). The question is how often can it be exceeded? This can be formulated two ways:

- in terms of frequency – how many times per certain time period?
- in terms of return period – what is the mean period of time between occurrences of such events?

In addition, the selection of R_n involves a considerable degree of subjective judgment. The actual role of the SLS of deflection is to provide adequate stiffness to the bridge as excessive vibration can lead to gradual deterioration and limitation of ability to carry the traffic. Therefore, the following procedure is being developed for SLS of deflection.

Step 1 Representative Components and Structures

Representative components and structures will be identified and selected to be considered in the development of code provisions for the SLS-Deflection.

Step 2 Load Model

For each considered component and structure, values of load components will be determined, including nominal values as well as the statistical parameters for loads as random variables. The parameters of time-varying loads will be determined for various time periods. The analysis will be performed for various traffic parameters (ADTT, legal loads, multiple presence, traffic patterns). The truck database will include the available sources including recent weigh-in-motion (WIM) data. The load frequencies will serve as a basis for determination of acceptability criteria in Step 4.

Step 3 Actual Deflections

For each considered component and structure, the actual deflections will be calculated using advanced (linear) finite element method (FEM). The calculations will be performed for single lane loaded, two adjacent lanes loaded, and more lanes loaded if applicable (see Fig.1 to Fig.3). The results will serve as a basis for the development of probability density functions (PDF) of deflection, representing values of deflection vs. frequency of occurrence. It is expected that these PDF's will be structure-specific and strongly site-specific.

The PDF type can be assumed as normal, and for each considered case, there will be two parameters defined: the mean (or bias factor) and coefficient of variation. Bias factor is the ratio of the mean-to-nominal value, and coefficient of variation is the ratio of standard deviation and the mean value.

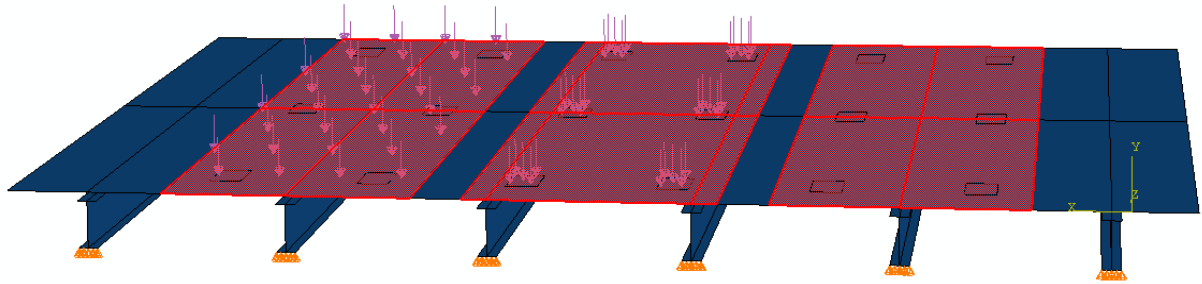


Fig. 1. Model of the Bridge

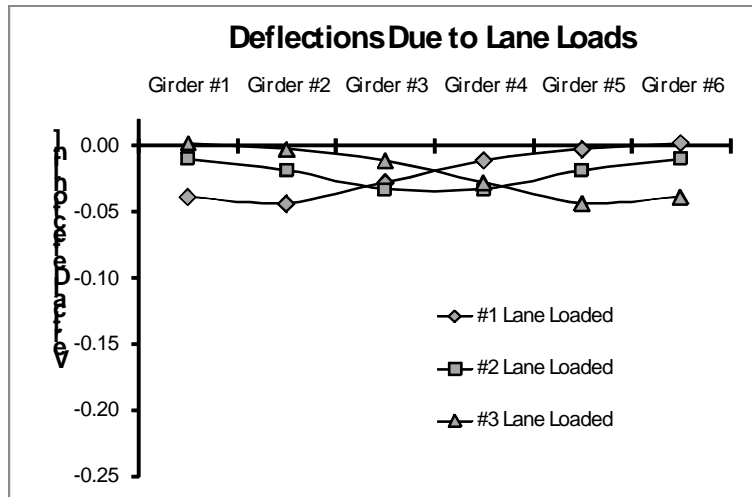


Fig. 2. Deflections at Midspan due to Lane Loads

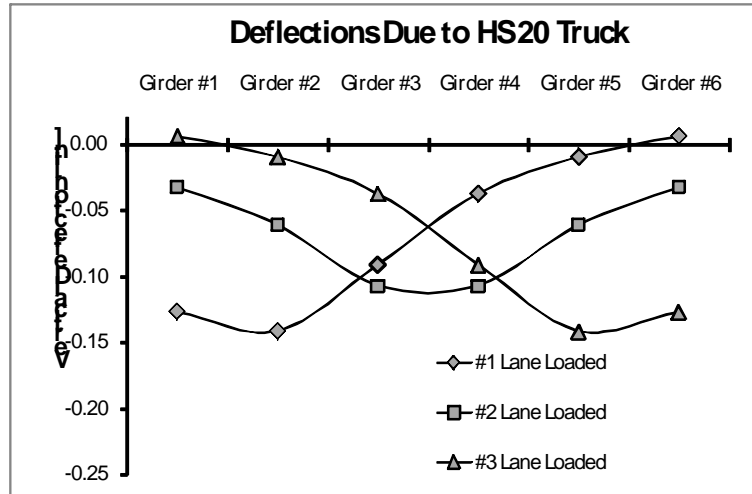


Fig. 3. Deflections at Midspan due to HS-20 Trucks

Step 4 Acceptability Criteria

Establishing the acceptability criteria and this means the selection of two items:

- The maximum allowable deflection limit for the considered representative components and structures.
- The minimum return period (or maximum frequency) for occurrence of the maximum allowable deflection limit.

This is the most difficult task in this calibration. The deflection limit is not only subjective (different expert can have a different opinion), but it is unrealistic to formulate the limit as a deterministic boundary. Instead, it is proposed to express the maximum allowable deflection

limit in terms of a probability density function (PDF) that is to be obtained by plotting the deflection limit vs. the corresponding allowable frequency of occurrence. The important is only the upper tail of the distribution (heavy trucks).

The PDF type can be assumed as normal, and for each considered case, there will be two parameters defined: the mean (or bias factor) and coefficient of variation. Bias factor is the ratio of the mean-to-nominal value, and coefficient of variation is the ratio of standard deviation and the mean value.

Step 5 Reliability Analysis

The limit state function is as shown in Eq. 2, with R representing the deflection limit (Fig. 1) and Q representing the load effect (deflection) frequency, determined in Step 3. The reliability indices can be determined using Monte Carlo method (Nowak and Collins 2000). Reliability indices will be calculated for a wide range of considered components and structures (identified in Step 1).

Step 6 Target Reliability Index for Deflection

The target reliability depends on:

- What are the consequences of exceeding the considered limit state? What happens when the deflection limit is exceeded too often?
- What is the cost of reducing the deflection limit? What are the gains of increasing the deflection limit?
- What is the reliability index corresponding to SLS of deflection in the current design code? The resulting reliability indices obtained in Step 5 will be reviewed and they will serve as a basis for the selection of the target reliability index for deflection.

Step 7 Load and Resistance Factors for Deflection

This is the final step of the calibration procedure. Various sets of load and resistance factors will be considered, taking into consideration:

- Closeness to the target reliability index
- User-friendly code format (reduced number of different load and resistance factors, simplified structural analysis)
- Minimized departure from the current practice
- Load and resistance factors rounded to the nearest 0.5.

As a final check, the reliability analysis will be performed for representative structures redesigned using the selected live load factor and resistance factor. The results will be plotted and compared with the reliability indices obtained prior to calibration.

5. Calibration for SLS-Decompression

The design of prestressed concrete bridge girders is usually governed by the tension limit at the bottom of the concrete girder. It is assumed that the girder will crack when a very heavy truck occurs on the bridge, and once concrete is cracked the crack opens each time decompression moment is exceeded. The reliability analysis is performed to determine the reliability index for SLS-decompression.

Pre-tensioned girder bridges with spans from 12m to 36m are considered, with six AASHTO types shown in Fig.4. The deck is assumed to be 200 mm thick with 1.8m spacing between the girders and with 75mm wearing surface. The girders are designed according to the AASHTO Code [1]. The analysis is limited to interior girders. Live load is HL93 loading,

which is a superposition of a 35 Ton truck and a uniformly distributed load of 9.3 kN/m. It is assumed that for SLS-decompression, no tensile stress is allowed in the girder. The girder distribution factors are taken from the AASHTO Code [1].

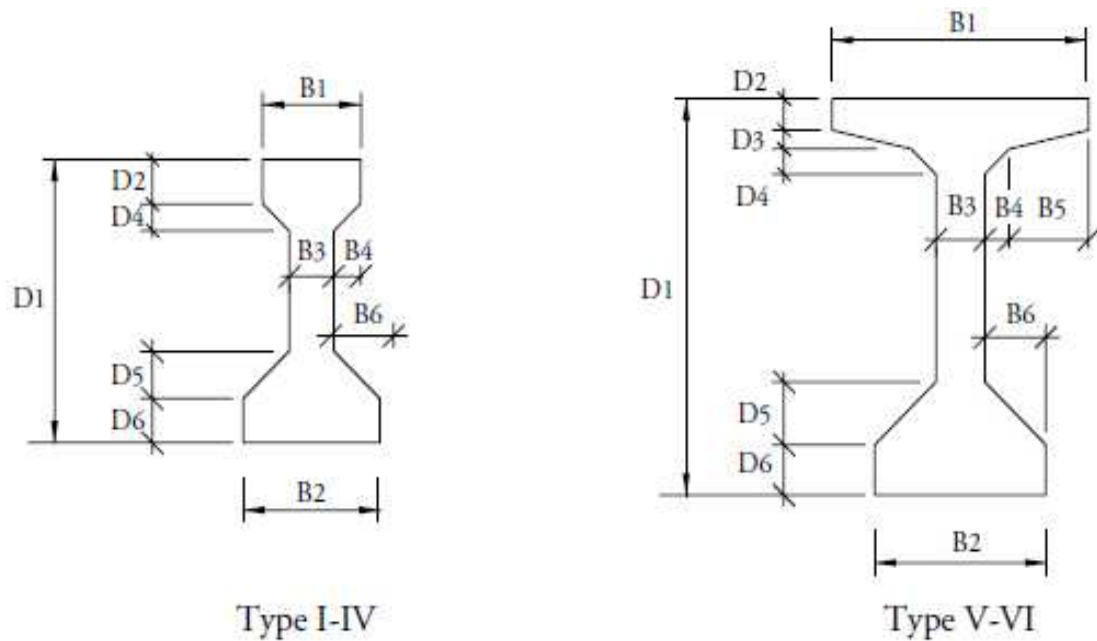


Fig. 4. AASHTO I-Girders [5]

For the considered girders, the decompression moment is simulated using Monte Carlo procedure. The resulting cumulative distribution functions (CDF) for the decompression moment are plotted on the normal probability paper (see Figures 6-9). Obtained CDF's are then analyzed together with the processed WIM data from Oregon to determine the number of decompressions.

The design formula specified by AASHTO (2007) for SLS-decompression is following:

$$U = 1.0(DC + DW) + 0.8(LL + IM) + 1.0WA + 1.0FR \quad (6)$$

where:

- DC – dead load effect
- DW – wearing surface load effect
- LL – live load effect
- IM – dynamic load allowance effect
- WA – water load and steam pressure load effect
- FR – friction load effect

First step in the design procedure is to determine total load demand. Dead load included the weight of the girder itself, weight of the slab and wearing surface. Live load effects are calculated based on HL93 loading:

$$M_{\max} = (M_{Truck} + M_{Lane} + 0.33M_{Truck}) \cdot GDF \quad (7)$$

$$M_{Truck} = 16 \left(\frac{9}{8}L + \frac{\alpha}{L}(21 - 4.5\alpha) - 17.5 \right) \quad (8)$$

$$M_{Lane} = 0.08(L^2 - 4\alpha^2) \quad (9)$$

$$\alpha = \frac{336}{144 + 0.64L} \quad (10)$$

$$GDF = 0.075 \left(\frac{S}{9.5} \right)^{0.6} \left(\frac{S}{L} \right)^{0.2} \left(\frac{K_g}{12Lt_s^3} \right) \quad (11)$$

Assumed theoretical model of the girder geometry and the stress distribution at the decompression stage is presented in the Figure 5. The design equation for the considered case can be expressed as follows:

$$0 = -\frac{P_f}{A_g} - \frac{P_f e_m y_{tg}}{I_g} + \frac{(M_{dg} + M_{ds}) y_{tg}}{I_g} + \frac{(M_{dw} + 0.8M_{LL}) y_{ic}}{I_c} \quad (12)$$

where:

- P_f – prestressing force after all losses,
- A_g – gross cross-section area,
- y_{tg} – distance from the center of gravity of the girder to the top fiber of the girder,
- I_g – moment of inertia of the girder,
- I_c – moment of inertia of the composite section,
- y_{ic} – distance from the center of the gravity of the composite section to the top fiber of the girder,
- M_{dg} – dead load moment due to the girder weight,
- M_{ds} – dead load moment due to the weight of the slab,
- M_{dw} – dead load moment due to the wearing surface,
- M_{LL} – dead load moment due to live load.

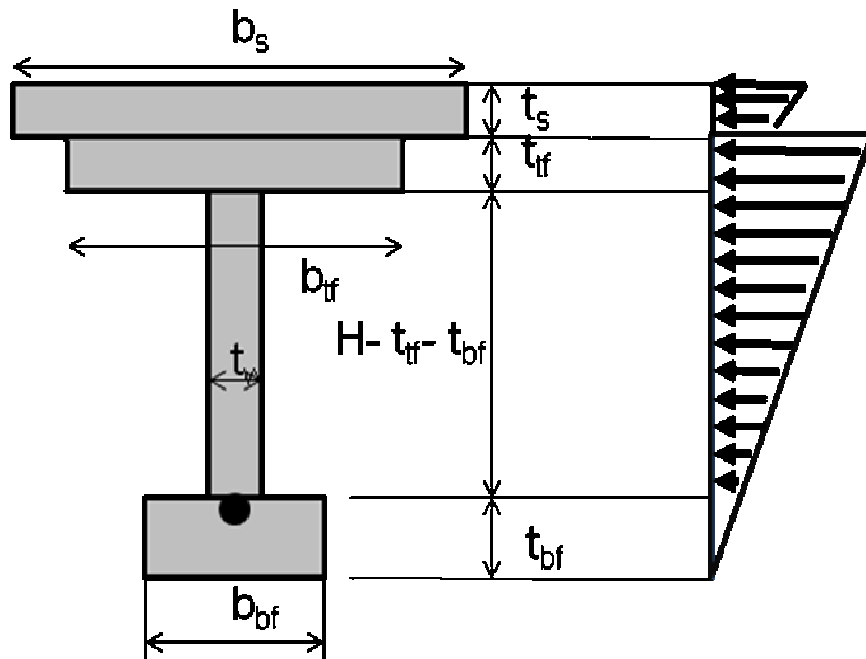


Fig. 5. Theoretical Model for Decompression Moment

The amount of necessary reinforcement was simply calculated by rearrangement of the equation 12:

$$A_{ps} = \frac{\frac{M_{dg} + M_{ds}}{S_{bg}} - \frac{M_{DW} + 0.8M_L}{S_{bc}}}{\frac{f_{ps}}{A_g} - \frac{f_{ps} \cdot e_m}{S_{bg}}} \quad (13)$$

Decompression moment can be expressed as:

$$M_{Dec} = \frac{P_f I_c}{A_g y_{ic}} + \frac{P_f e_m y_{tg} I_c}{I_g y_{ic}} - \frac{(M_{dg} + M_{ds}) y_{tg} I_c}{I_g y_{ic}} - M_{dw} \quad (14)$$

Decompression moments were simulated with the Mote Carlo procedure for all considered bridges. Table presents the statistical parameters obtained from the analyses.

Table 1: Statistical Parameters of the Decompression Moment

Type	Span [ft]	Bias	V
I	40	1.25	0.11
II	60	1.28	0.12
III	80	1.31	0.14
IV	90	1.35	0.15
V	100	1.36	0.15
VI	120	1.40	0.17

New traffic data, was used to generate the cumulative distribution functions of the live loads. Previously calculated moment ratios were multiplied by the moment due to the HL93 load corrected for two lanes by girder distribution factor and plotted on normal probability paper together with the distributions of decompression moments. Figures 6 to 9 present CDF's of live load and decompression moments plotted together on normal probability paper.

From the generated CDFs the values of standard normal variables were found corresponding to different bending moments, depending on the span length and the type of the girder. Table summarizes all the ranges of moments included in the analysis.

Here it needs to be clearly stated, that despite the fact that both distributions are shown in one figure, and both are plotted versus standard normal variable, the interpretation and meaning of the vertical coordinate is different for each distribution. As mentioned before, the relationship between the standard normal variable and the probability is following:

$$\Phi(Z) = p \quad (15)$$

where p is probability and $\Phi(x)$ is the cumulative distribution function of the standard normal distribution. In case of CDF for the decompression moment, the i-th pair of $Z(i)$ and $M(i)$ is interpreted as the probability that the decompression moments will be smaller than $M(i)$. This distribution expresses the variability of the girder resistance due to material properties, dead loads, prestressing force and it is time independent. The statistical parameters depend on the design, workmanship and the quality of materials. The cumulative distribution function of

the live load effect, in this case maximum bending, characterizes time dependent variability of the traffic load. The $Z(i)$ and $M(i)$ pairs are interpreted as the probabilities of occurrence of a truck generating moment that is smaller or equal to $M(i)$. Knowing the truck volume (ADTT) on the particular road it is then possible to estimate how many trucks will generate moment that exceeds $M(i)$ in the investigated return period.

Table 2: Considered Moment Intervals.

Girder Type	Span [ft]	M [k-ft]	Step [f-ft]
I	40	300–800	5
II	60	450–1000	5
III	80	450–1500	5
IV	90	550–2000	10
V	100	550–2320	10
VI	120	550–2680	10

In this study it was decided to use a one month return period. For each moment interval the number of trucks heavier than $M(i)$ was calculated as:

$$N = p \cdot 30 \cdot ADTT \tag{16}$$

where p is the probability of occurrence of a truck causing moment that is higher than $M(i)$. Five different average daily truck traffic volumes were assumed for the analyses 1000, 2000, 3000, 4000 and 5000 trucks. Numbers of trucks were plotted versus the probability of occurrence of decompression moment not less than $M(i)$. In other words the vertical coordinate indicates the probability that the number of decompressions is less or equal to the number specified by the horizontal coordinate. Figures 12 to 17 present the resulting curves for all the analyzed girders and spans.

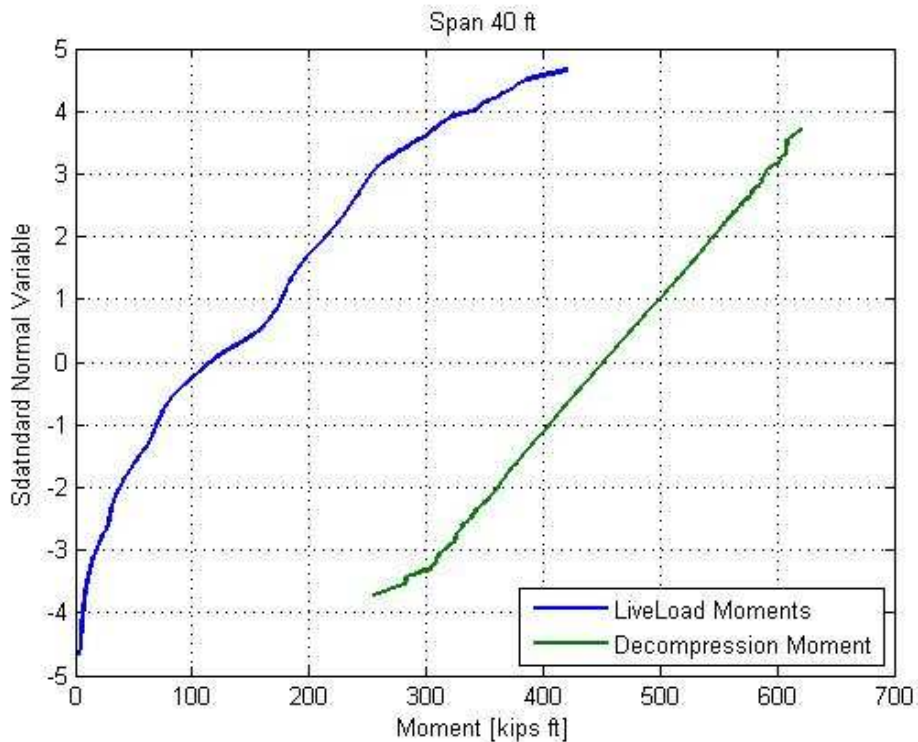


Fig. 6. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 40 ft

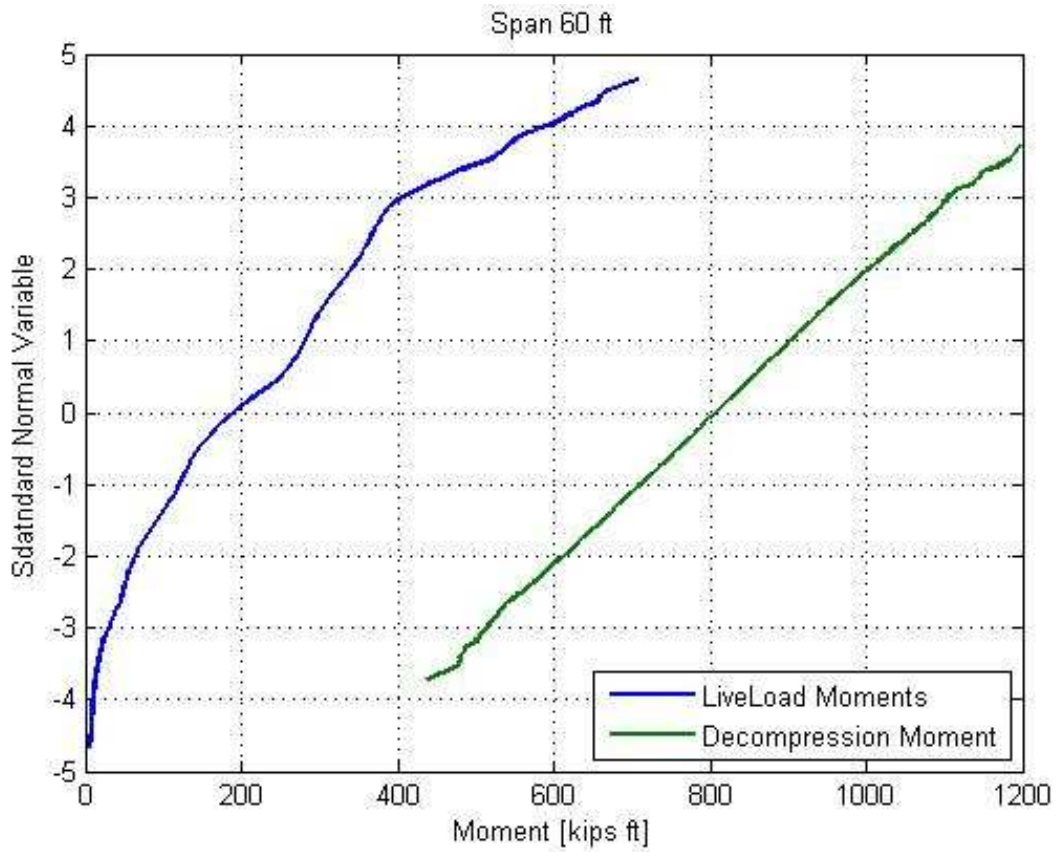


Fig. 7. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 60 ft

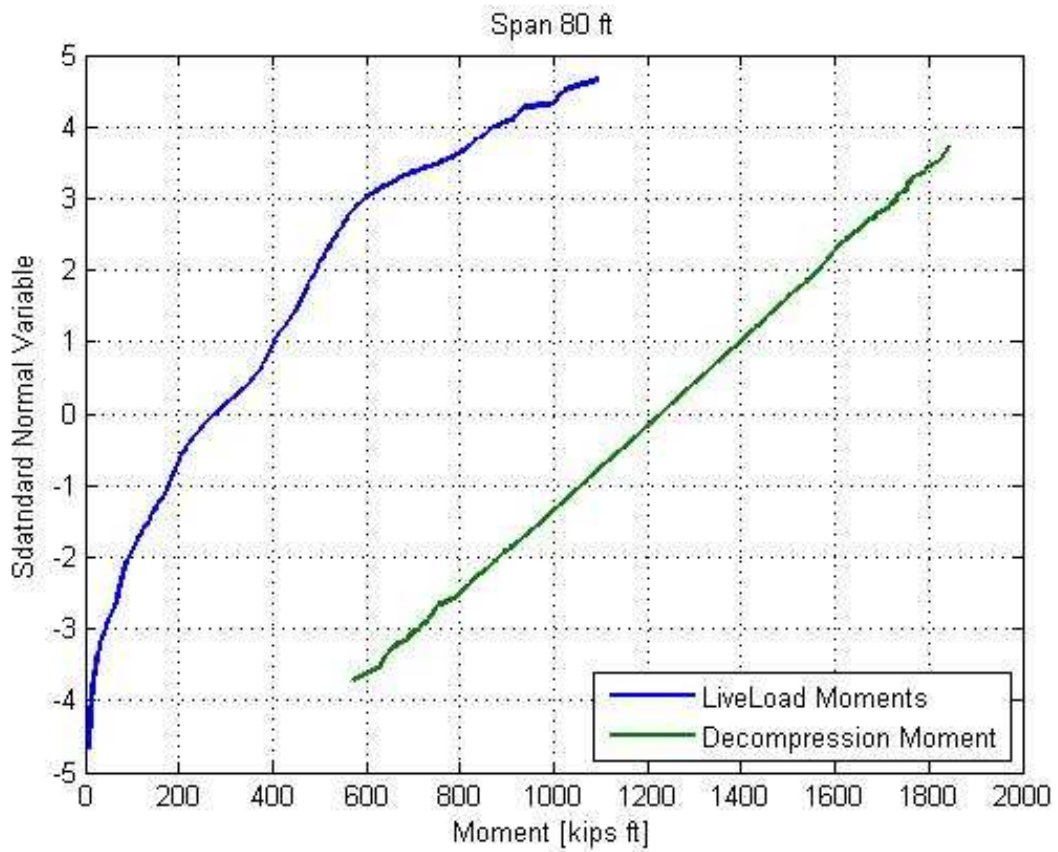


Fig. 8. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 80 ft

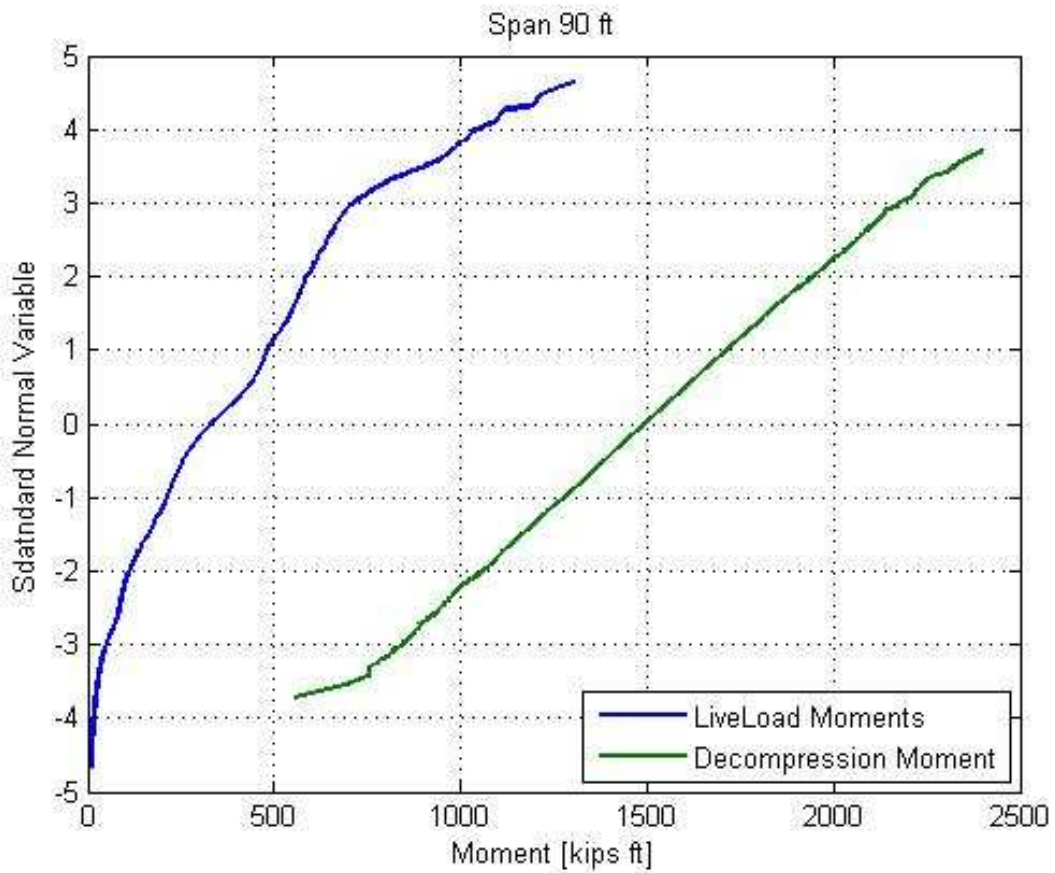


Fig. 9. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 90 ft

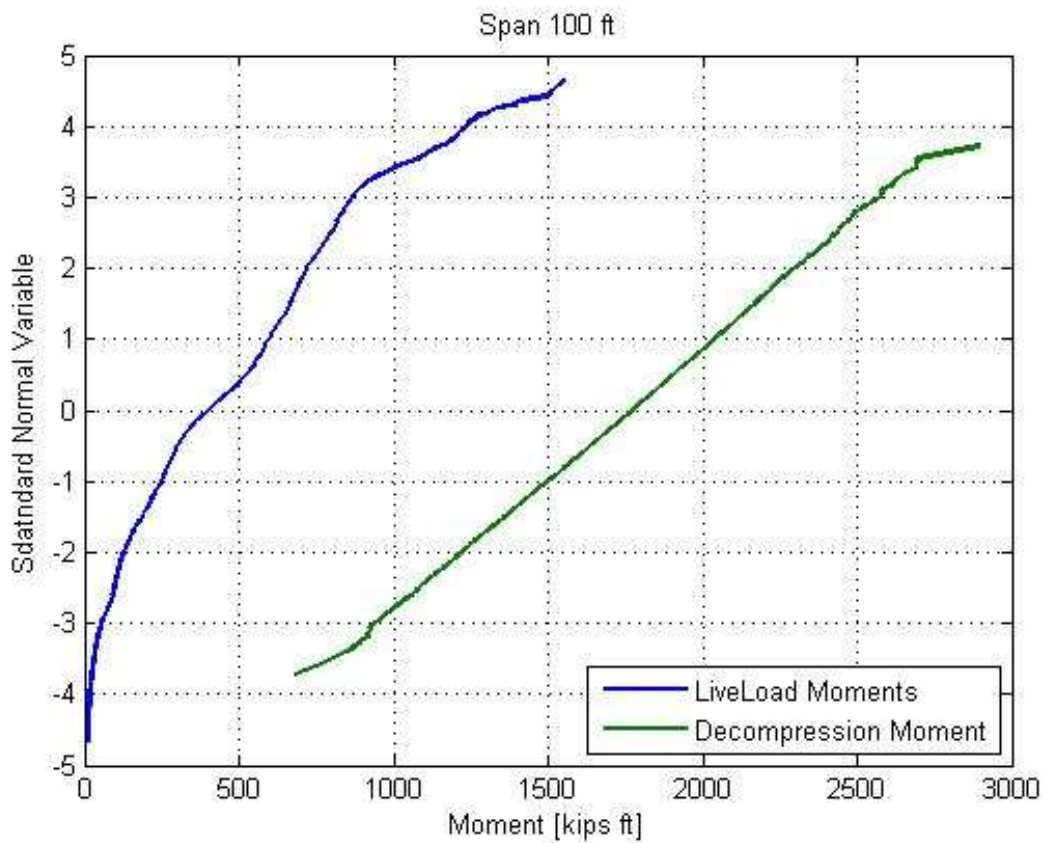


Fig. 10. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 100 ft

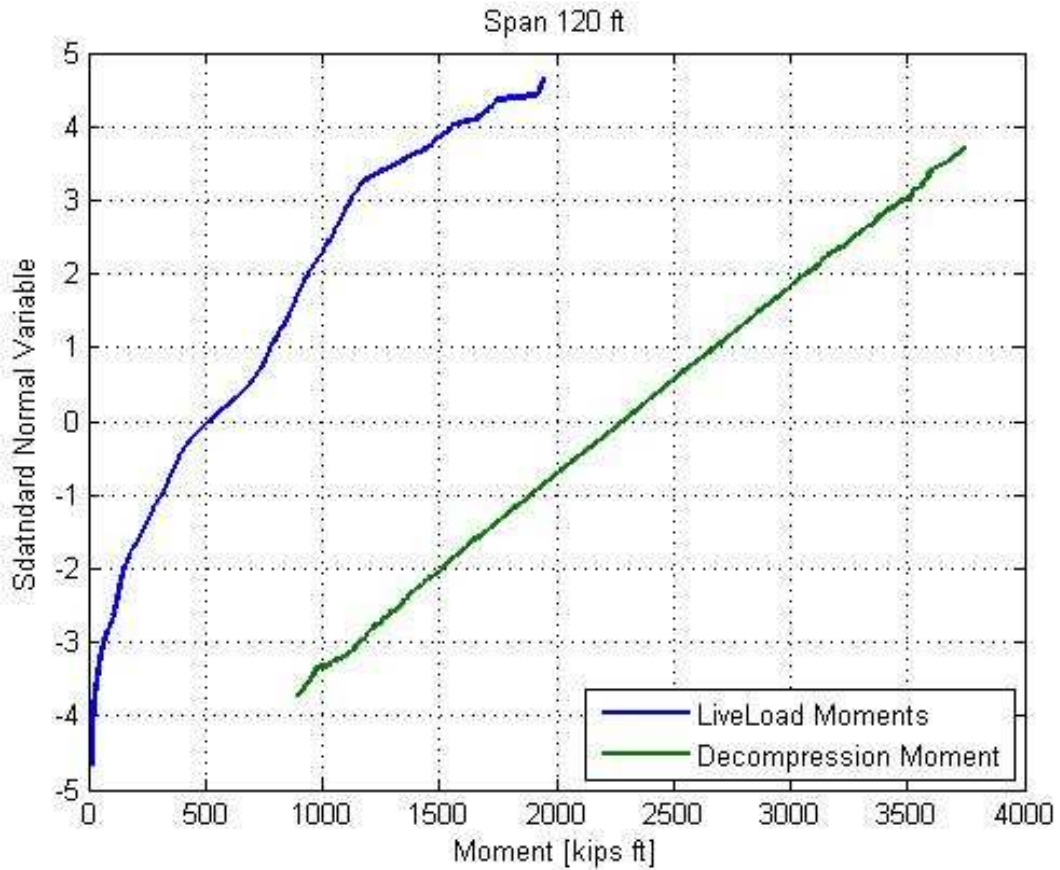


Fig. 11. Cumulative Distribution Functions for Live Load and Decompression Moments – Span 120 ft

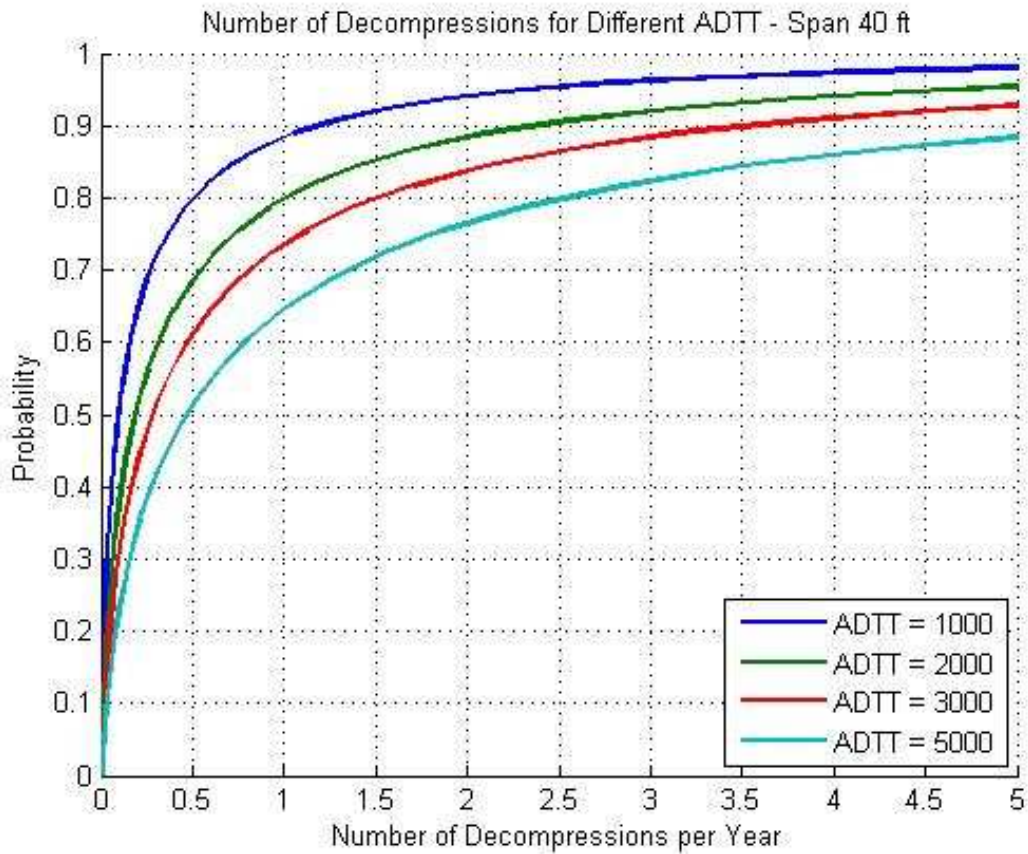


Fig. 12. Number of Decompressions vs. Reliability Index for Different ADTT – Span 40 ft

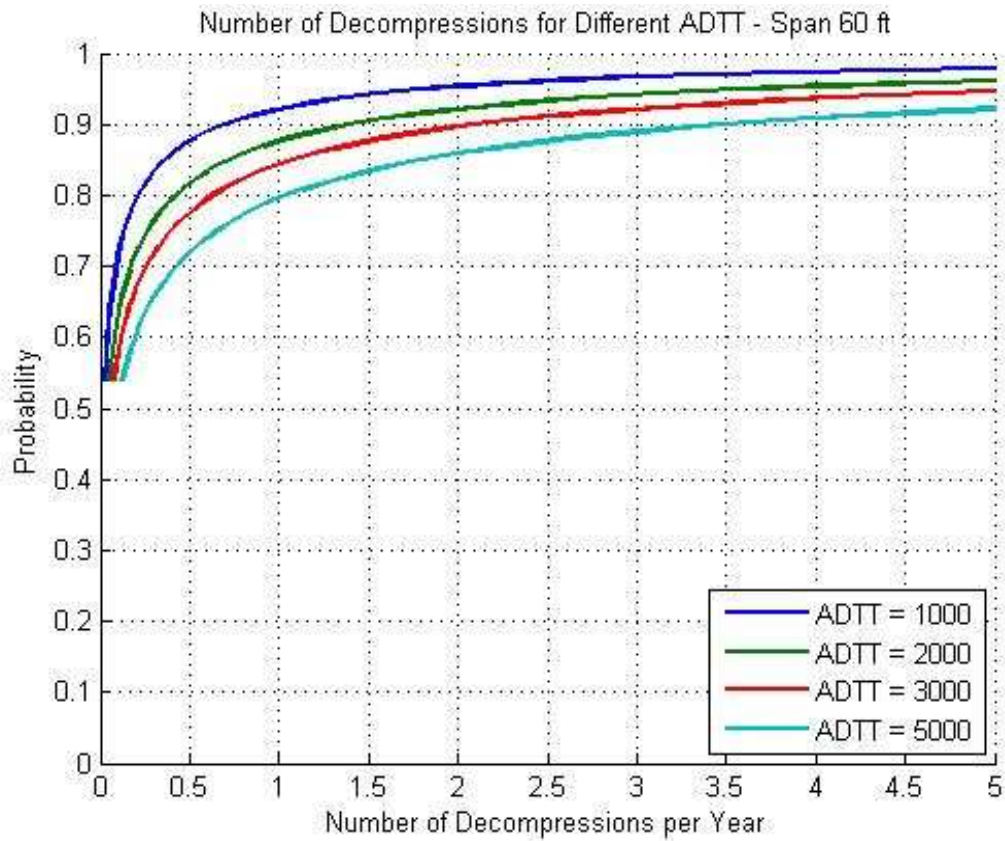


Fig. 13. Number of Decompressions vs. Reliability Index for Different ADTT – Span 60 ft

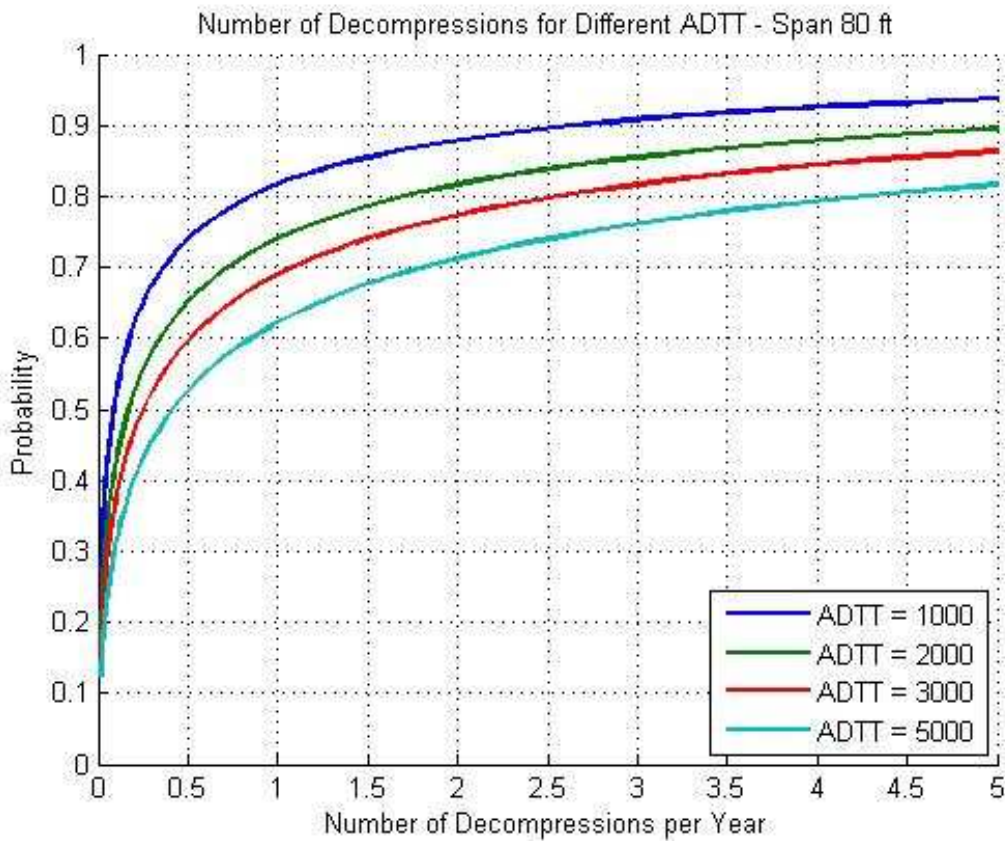


Fig. 14. Number of Decompressions vs. Reliability Index for Different ADTT – Span 80 ft

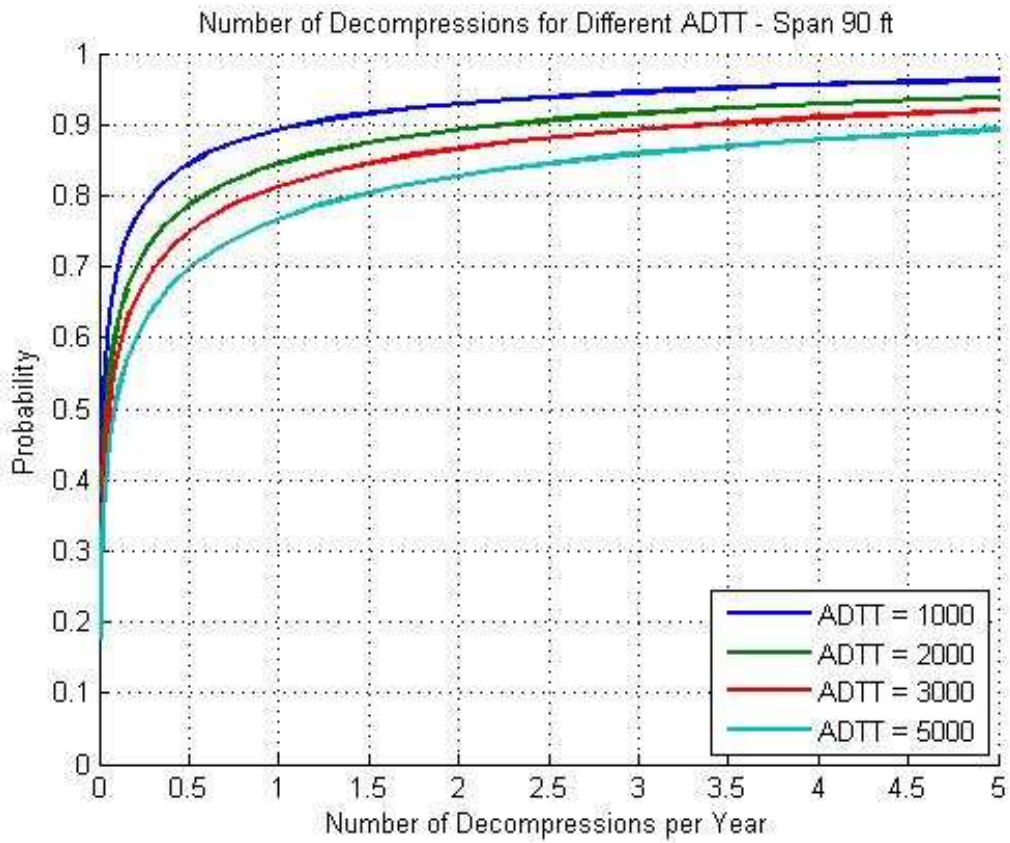


Fig. 15. Number of Decompressions vs. Reliability Index for Different ADTT – Span 90 ft

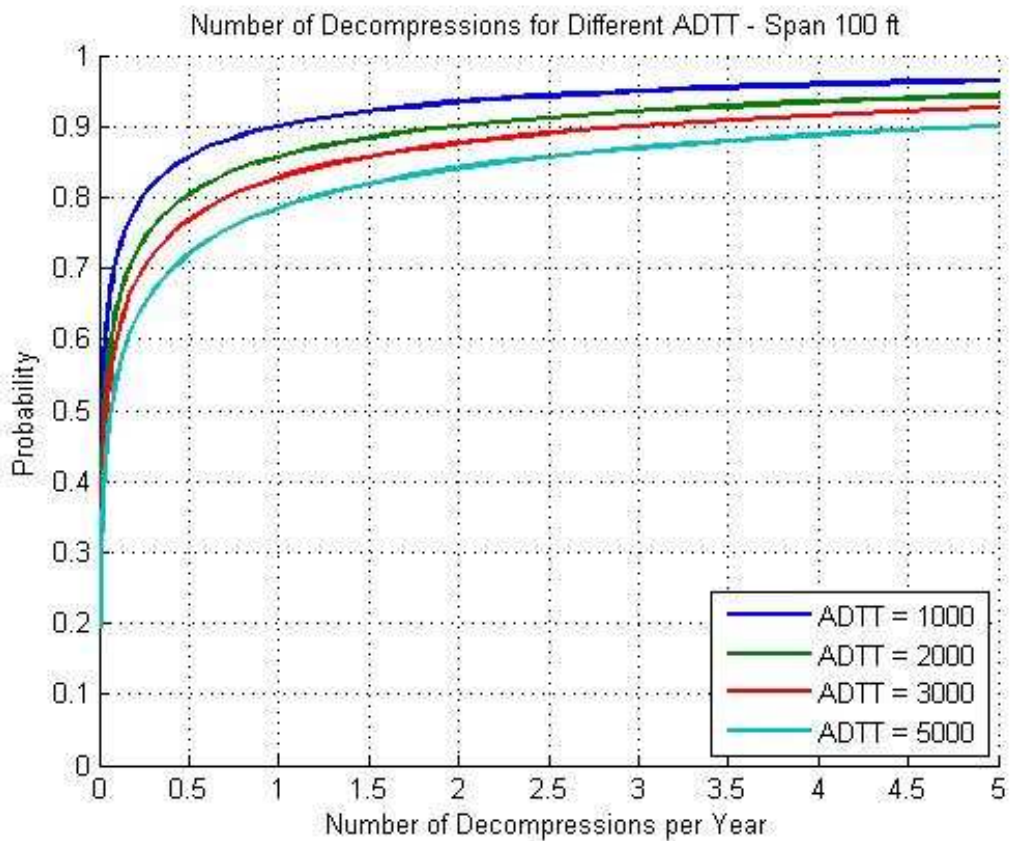


Fig. 16. Number of Decompressions vs. Reliability Index for Different ADTT – Span 100 ft

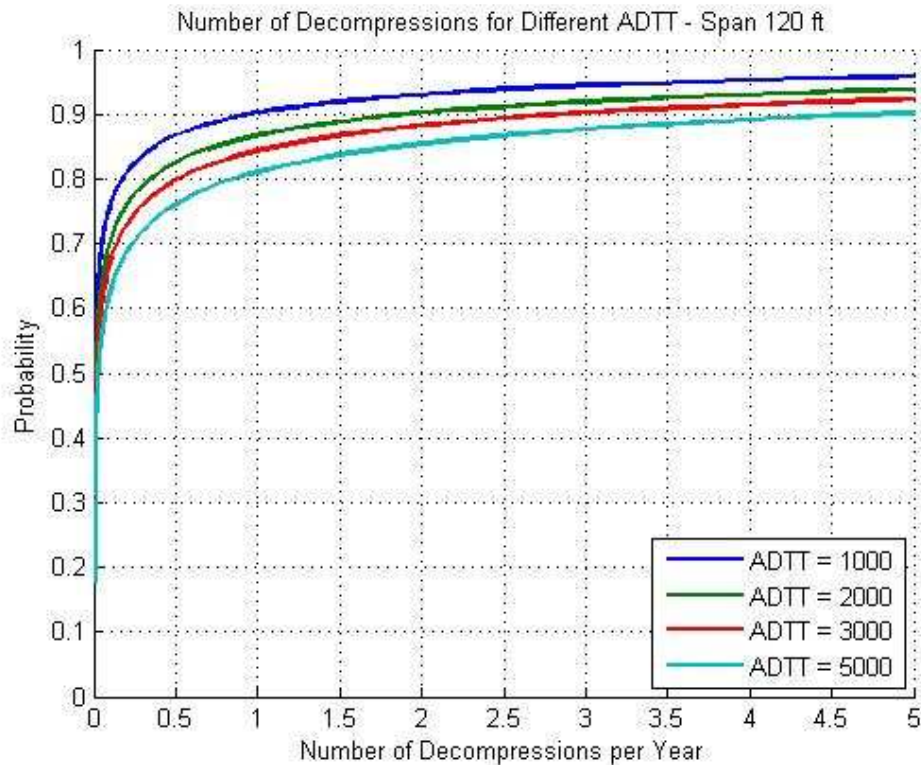


Fig. 17. Number of Decompressions vs. Reliability Index for Different ADTT – Span 120 ft

5. Conclusions

The approach to ultimate limit states and serviceability limit states is considered. The new generation of design codes is developed in the reliability-based calibration process, which has been focused on the ULS so far. There is now a growing need to develop calibration procedures for SLS.

The major difference is formulation of the limit state function for ULS and SLS is the difficulty of selecting the acceptability criteria for SLS. These criteria are usually very subjective.

There are considerable differences in consequences of exceeding the limit state between ULS and SLS. Therefore, the target reliability indices are also very different. For ULS, the target β can be about 3–4, while for SLS they can be as low as 0.

In the new generation of design codes, the load and resistance factors are determined in the reliability-based calibration procedures. The calibration procedures for ULS is available in literature but not for SLS. Therefore, a procedure is presented for SLS-deflection and SLS-decompression.

The reliability analysis for SLS-decompression is demonstrated on typical pre-tensioned concrete girders.

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